Journal of Engineering Physics and Thermophysics, Vol. 74, No. 1, 2001

MOMENTUM DEFECT IN IMPACT OF A SHORT ROD AGAINST A SMOOTHOBSTACLE

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UDC 539.3

The problem of deformation of an elastic half-plane under the effect of a finite load of special form on its boundary is studied theoretically. Using Laplace and Fourier transformations the authors obtained a finite relation for the total force on the axis of symmetry of the half-plane that is used to study momentum loss in impact of a short elastic rod against a smooth absolutely rigid obstacle. With certain limitations on the dimensions of the rod the authors obtained an accurate analytical estimate of the momentum loss that is associated with conversion of part of the energy of the striker to the energy of its transverse oscillations.

1. We consider the plane dynamic problem of deformation of an elastic half-plane ($0 \le x < +\infty$, $-\infty < y < +\infty$) under the effect, on its boundary ($x = 0$, $-\infty < y < +\infty$), of a load that depends on time and having the following form (Fig. 1a):

$$\sigma_{xy} = 0 \quad \text{when} \quad x = 0 \,, \quad -\infty < y < +\infty \,,$$

$$\sigma_{xx} = 0 \quad \text{when} \quad x = 0 \,, \quad |y| > c_1 t \,,$$

$$\sigma_{xx} = \sigma_0 \quad \text{when} \quad x = 0 \,, \quad |y| < c_1 t \,, \tag{1}$$

$$c_1 = [(\lambda + 2\mu)/\rho]^{1/2} \,.$$

We note that the dynamic problem of investigation of the stress–strain state of an elastic half-plane under the action of different loads at its boundary has been considered in many works, for example, [1–3]. The objective of the solution presented in this paper is just to obtain certain integral characteristics which are used in calculation of the momentum defect.

The equations of motion for the half-plane are written in the form

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2} \,, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \,, \tag{2}$$

The initial conditions are as follows:

$$u = v = 0 \,; \quad \frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0 \quad \text{when} \quad t = 0 \,, \quad x > 0 \,. \tag{2'}$$

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1062-0125/01/7401-0158$25.00

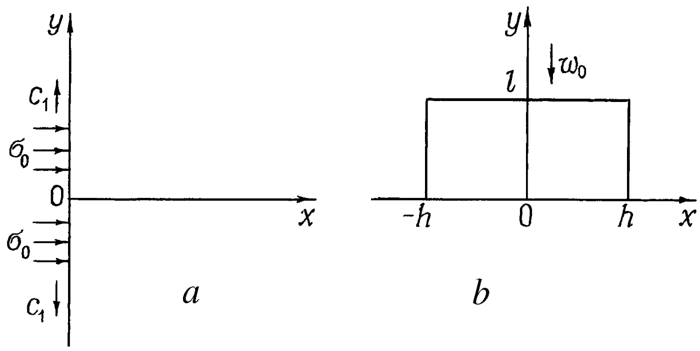

Fig. 1. Computational regions for an elastic half-plane ($0 \le x < +\infty$, $-\infty < y < +\infty$) under the effect of a load of special form on the boundary ($x = 0$, $-\infty < y < +\infty$) (a) and for an elastic rod ($-h \le x \le h$, $0 \le y \le l$) in impact against an absolutely rigid half-plane ($-\infty < x < +\infty$, $-\infty < y \le 0$) (b).

The boundary conditions are relations (1). The relationship between stresses and strains is defined by an ordinary elastic law:

$$\sigma_{xx} = \lambda\theta + 2\mu\varepsilon_{xx}, \quad \sigma_{yy} = \lambda\theta + 2\mu\varepsilon_{yy}, \quad \sigma_{xy} = 2\mu\varepsilon_{xy}, \tag{3}$$

where

$$\theta = \varepsilon_{xx} + \varepsilon_{yy}; \quad \varepsilon_{xx} = \frac{\partial u}{\partial x}; \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}; \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right);$$

$$c_2 = (\mu/\rho)^{1/2}; \quad c = c_2/c_1 .$$

According to (2) and (3), we have

$$c_1^2 \frac{\partial^2 u}{\partial x^2} + c_1^2 c^2 \frac{\partial^2 u}{\partial y^2} + c_1^2 (1 - c^2) \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 u}{\partial t^2}, \quad c_1^2 (1 - c^2) \frac{\partial^2 u}{\partial x \partial y} + c_1^2 c^2 \frac{\partial^2 v}{\partial x^2} + c_1^2 \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial t^2}. \tag{4}$$

Applying the Laplace transformation with respect to t ((p) = $\left(f^*(p) = \int_0^\infty f(t) \exp(-pt)\, dt\right)$ to relations (4) and boundary conditions (1), we obtain the equations of motion

$$c_1^2 \frac{\partial^2 u^*}{\partial x^2} + c_1^2 c^2 \frac{\partial^2 u^*}{\partial y^2} + c_1^2 (1 - c^2) \frac{\partial^2 v^*}{\partial x \partial y} = p^2 u^*,$$

$$c_1^2 (1 - c^2) \frac{\partial^2 u^*}{\partial x \partial y} + c_1^2 c^2 \frac{\partial^2 v^*}{\partial x^2} + c_1^2 \frac{\partial^2 v^*}{\partial y^2} = p^2 v^* \tag{5}$$

with the boundary conditions at $x = 0$

$$\frac{\partial u^*}{\partial y} + \frac{\partial v^*}{\partial x} = 0, \quad c_1^2 \frac{\partial u^*}{\partial x} + c_1^2 (1 - 2c^2) \frac{\partial v^*}{\partial y} = \frac{\sigma_0}{\rho} \frac{1}{p} \exp\left(-p \frac{|y|}{c_1}\right), \tag{6}$$

where u^* and v^* are the transforms of the functions u and v in Laplace transformation with respect to t.

We apply the Fourier transformation with respect to y to Eqs. (5) and (6):

$$f^0(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y) \exp(-i\omega y)\, dy .$$

We obtain the equations of motion in the form

$$c_1^2 \frac{\partial^2 u^0}{\partial x^2} - \omega^2 c_1^2 c^2 u^0 + c_1^2 (1 - c^2)\, i\omega \frac{\partial v^0}{\partial x} = p^2 u^0 , \quad c_1^2 (1 - c^2)\, i\omega \frac{\partial u^0}{\partial x} + c_1^2 c^2 \frac{\partial^2 v^0}{\partial x^2} - c^2 \omega^2 v^0 = p^2 v^0 , \tag{7}$$

with the boundary conditions at $x = 0$

$$i\omega u^0 + \frac{\partial v^0}{\partial x} = 0 , \quad c_1^2 \frac{\partial u^0}{\partial x} + c_1^2 (1 - 2c^2)\, i\omega v^0 = \frac{\sigma_0}{\rho} \sqrt{\frac{2}{\pi}} \frac{c_1}{p^2 + c_1^2 \omega^2} , \tag{8}$$

where u^0 and v^0 are the Fourier transforms of the functions u^* and v^*, respectively; i is the imaginary unit.

Under the assumption that the solution for $x \to \infty$ is bounded, the solution of (7) is written in the form

$$\begin{pmatrix} u^0 \\ v^0 \end{pmatrix} = D_1 \exp(-\lambda_1 x) \begin{pmatrix} i\lambda_1 \\ \omega \end{pmatrix} + D_2 \exp(-\lambda_2 x) \begin{pmatrix} \omega \\ -i\lambda_2 \end{pmatrix}, \tag{9}$$

where

$$\lambda_1^2 = \frac{p^2}{c_1^2} + \omega^2 ; \quad \lambda_2^2 = \frac{p^2}{c_1^2 c^2} + \omega^2 .$$

The first term corresponds to the potential component of the solution, while the second term corresponds to the vortical (solenoidal) component.

Using boundary conditions (8), we obtain

$$D_1 = \frac{\sigma_0}{\rho} \sqrt{\frac{2}{\pi}} \frac{c_1}{p^2 + c_1^2 \omega^2} \frac{i(\omega^2 + \lambda_2^2)\, c^2 c_1^2}{\Delta} , \quad D_2 = \frac{\sigma_0}{\rho} \sqrt{\frac{2}{\pi}} \frac{c_1}{p^2 + c_1^2 \omega^2} \frac{2\omega\lambda_1 c^2 c_1^2}{\Delta} ,$$

$$\Delta = (p^2 + 2c^2 c_1^2 \omega^2) - 4c^4 c_1^4 \omega^2 \lambda_1 \lambda_2 . \tag{10}$$

From (3) and (9), we find

$$\sigma_{yy}^0 = c_1^2 \rho D_1 \exp(-\lambda_1 x) [\omega^2 - (1 - 2c^2) \lambda_1^2]\, i + c_1^2 \rho D_2 \exp(-\lambda_2 x) (2c^2 \omega \lambda_2) . \tag{11}$$

Using (10) and (11), we have

$$\sigma_{yy}^0 = \sigma_0 \sqrt{\frac{2}{\pi}} \frac{c_1^5 c^2}{(p^2 + c_1^2 \omega^2)} \frac{1}{\Delta} \Big\{ -(\omega^2 + \lambda_2^2) [\omega^2 - (1 - 2c^2) \lambda_1^2] \exp(-\lambda_1 x) +$$

$$+ 4c^2 \omega^2 \lambda_1 \lambda_2 \exp(-\lambda_2 x) \Big\} . \tag{12}$$

The total force in the direction of the y axis at the cross section y = const is

$$f(y, t) = \int_0^\infty \sigma_{yy}(x, y, t)\, dx .$$

Then, according to (11):

$$f^0(\omega, p) = \sigma_0 \sqrt{\frac{2}{\pi}} \frac{c_1^2 (1 - 2c^2)}{(p^2 + c_1^2 \omega^2)^{3/2}} \frac{p^4}{\Delta} . \tag{13}$$

Since $f^0(\omega, p)$ is an even function of ω,

$$f^*(y, p) = \int_0^\infty \sqrt{\frac{2}{\pi}} f^0(\omega, p) \cos \omega y d\omega .$$

At $y = 0$ we have

$$f^*(0; p) = \frac{2}{\pi} \sigma_0 c_1^2 (1 - 2c^2) \int_0^\infty \frac{p^4}{(p^2 + c_1^2 \omega^2)^{3/2}} \times$$

$$\times \frac{1}{(p^2 + 2c^2 c_1^2 \omega^2)^2 - 4c^4 c_1^4 \omega^2 \sqrt{\dfrac{p^2}{c_1^2} + \omega^2} \sqrt{\dfrac{p^2}{c^2 c_1^2} + \omega^2}} d\omega .$$

Having substituted the variables $z = p/c_1\omega$, we obtain

$$f(0; p) = \frac{2}{\pi} \frac{c_1 (1 - 2c^2)}{p^2} \sigma_0 \int_0^\infty \frac{z^5}{(z^2 + 1)^{3/2}} \frac{dz}{(z^2 + 2c^2)^2 - 4c^3 \sqrt{z^2 + 1} \sqrt{z^2 + c^2}} .$$

Then

$$f(0; t) = \frac{2}{\pi} c_1 (1 - 2c^2) \sigma_0 k(c) t , \tag{14}$$

where

$$k(c) = \int_0^\infty \frac{z^5}{(z^2 + 1)^{3/2}} \frac{dz}{(z^2 + 2c^2)^2 - 4c^3 \sqrt{z^2 + 1} \sqrt{z^2 + c^2}} . \tag{15}$$

We note that $k(c) = 1$ when $c = 0$.

We have the following relation for the stresses $\sigma^*_{yy}(x, 0, p)$:

$$\sigma^*_{yy}(x, 0, p) = \frac{2}{\pi} \sigma_0 c_1^5 c^2 \int_0^\infty \left[-\left(2\omega^2 + \frac{p^2}{c^2 c_1^2}\right)\left(2c^2\omega^2 - \frac{1 - 2c^2}{c_1^2} p^2\right) \times \right.$$

$$\times \exp\left(-x\sqrt{\frac{p^2}{c_1^2}+\omega^2}\right) + 4c^2\omega^2\sqrt{\frac{p^2}{c_1^2}+\omega^2}\sqrt{\frac{p^2}{c_1^2c^2}+\omega^2}\times$$

$$\left.\times \exp\left(-x\sqrt{\frac{p^2}{c_1^2c^2}+\omega^2}\right)\right]\times$$

$$\times\frac{1}{(p^2+2c^2c_1^2\omega^2)^2-4c^4c_1^4\omega^2\sqrt{\frac{p^2}{c_1^2}+\omega^2}\sqrt{\frac{p^2}{c^2c_1^2}+\omega^2}}\frac{d\omega}{p^2+c_1^2\omega^2}.$$

Having substituted the variables $\omega = zp/c_1$, we obtain

$$\sigma_{yy}^*(x, 0, p) = \frac{2}{\pi}\sigma_0\int_0^\infty\left\{-(2c^2z^2+1)\left[2c^2z^2-(1-2c^2)\right]\frac{1}{p}\exp\left(-x\frac{p}{c_1}\sqrt{1+z^2}\right)+\right.$$

$$\left.+4c^3z^2\sqrt{1+z^2}\sqrt{1+c^2z^2}\,\frac{1}{p}\exp\left(-x\frac{p}{cc_1}\sqrt{1+c^2z^2}\right)\right\}\times$$

$$\times\frac{1}{(1+2c^2z^2)^2-4c^3z^2\sqrt{1+z^2}\sqrt{1+c^2z^2}}\frac{dz}{1+z^2}.$$

Then

$$\sigma_{yy}(x, 0, t) = \sigma_{yy1}(x, 0, t) + \sigma_{yy2}(x, 0, t),$$

where $\sigma_{yy1}(x, 0, t) = 0$ when $x > c_1t$ and

$$\sigma_{yy1}(x, 0, t) = \frac{2}{\pi}\sigma_0\int_0^{\sqrt{(c_1^2t^2)/x^2-1}}\frac{[1-2c^2-2c^2z^2]\,[1+2c^2z^2]}{[1+2c^2z^2]^2-4c^3z^2\sqrt{1+z^2}\sqrt{1+c^2z^2}}\frac{dz}{1+z^2} \tag{16}$$

when $x < c_1t$;

$$\sigma_{yy2}(x, 0, t) = 0$$

when $x > cc_1t$ and

$$\sigma_{yy2}(x, 0, t) = \frac{2}{\pi}\sigma_0\int_0^{\sqrt{(c_1^2t^2)/x^2-1/c^2}}\frac{4c^3z^2\sqrt{1+z^2}\sqrt{1+c^2z^2}}{[1+2c^2z^2]^2-4c^3z^2\sqrt{1+z^2}\sqrt{1+c^2z^2}}\frac{dz}{1+z^2} \tag{17}$$

when $x < cc_1t$.

If we substitute $z = \sqrt{1-u^2}/u$ in relation (15), then for $k(c)$ we obtain the expression

$$k(c) = \int_0^1 \frac{(1-u^2)\,[[(1-u^2) + 2c^2u^2]^2 + 4c^3u^2\sqrt{(1-u^2)+c^2u^2}\,]}{(1-u^2)^3 + 8c^2(1-u^2)^2u^2 + 8c^4(3-2c^2)(1-u^2)u^4 + 16c^6(1-c^2)u^6}\,du\,.$$

With a similar substitution in (16) we have

$$\sigma_{yy1}(x, 0, t) = \frac{2}{\pi}\sigma_0 \int_{x/(c_1t)}^{1} \frac{[u^2 - 2c^2]\,[u^2(1-2c^2) + 2c^2]}{[u^2(1-2c^2)+2c^2]^2 - 4c^3(1-u^2)\sqrt{u^2(1-c^2)+c^2}}\,\frac{du}{\sqrt{1-u^2}}\,.$$

Substitution of $z = \sqrt{(c^2 - u^2)}/cu$ in (17) yields

$$\sigma_{yy2}(x, 0, t) = \frac{2}{\pi}\sigma_0 \int_{x/(c_1t)}^{c} \frac{4c^4\sqrt{c^2-u^2}}{[2c^2-u^2]^2 - 4c^2(c^2-u^2)\sqrt{c^2-(1-c^2)u^2}}\,\frac{du}{\sqrt{c^2-(1-c^2)u^2}}\,.$$

2. We use the results obtained to estimate the momentum defect in impact of an elastic rod against a smooth absolutely rigid obstacle.

In the region $-h \le x \le h$, $0 \le y \le l$ (see Fig. 1b) we have Eqs. (2) with the initial conditions

$$u = v = 0\,, \quad \frac{\partial u}{\partial t} = 0\,, \quad \frac{\partial v}{\partial t} = -w_0 \quad \text{at} \quad t = 0$$

and the boundary conditions

$$\sigma_{xx} = 0\,, \quad \sigma_{xy} = 0 \quad \text{when} \quad x = \pm h\,, \quad 0 < y < l\,,$$

$$\sigma_{yy} = 0\,, \quad \sigma_{xy} = 0 \quad \text{when} \quad -h \le x \le h\,, \quad y = l\,,$$

$$\sigma_{xy} = 0\,, \quad v = 0 \quad \text{when} \quad -h \le x \le h\,, \quad y = 0\,.$$

We represent the solution as

$$u(x, y, t) = u_0(y, t) + u_1(x, y, t)\,, \quad v(x, y, t) = v_0(y, t) + v_1(x, y, t)\,,$$

where $u_0(y, t) = 0$ and $v_0(y, t)$ satisfies the equation

$$(\lambda + 2\mu)\frac{\partial^2 v_0}{\partial y^2} = \rho\frac{\partial^2 v_0}{\partial t^2}$$

with the initial conditions

$$v_0 = 0\,, \quad \frac{\partial v_0}{\partial t} = -w_0 \quad \text{at} \quad t = 0$$

and the boundary conditions

$$v_0 = 0 \quad \text{at} \quad y = 0\,, \quad \frac{\partial v_0}{\partial y} = 0 \quad \text{at} \quad y = l\,.$$

We introduce the dimensionless variables

$$y' = \frac{y}{l}, \quad x' = \frac{x}{l}, \quad t' = \frac{tc_1}{l}, \quad u' = \frac{u}{l}, \quad v' = \frac{v}{l}.$$

In the dimensionless variables when $0 < t' < 1$ (i.e., when $0 < t < l/c_1$)

$$v_0' (y', t') = \begin{cases} -w_0' t' & \text{when} \quad t' < y' < 1, \\ -w_0' y' & \text{when} \quad 0 < y' < t'. \end{cases}$$

When $1 < t' < 2$ (i.e., when $l/c_1 < t < 2l/c_1$)

$$v_0' (y', t') = \begin{cases} -w_0' (-t' + 2) & \text{when} \quad 2 - t' < y' < 1, \\ -w_0' y' & \text{when} \quad 0 < y' < 2 - t', \end{cases}$$

here $w_0' = w_0/c_1$. Then the functions u_1 and v_1 satisfy Eqs. (4), initial conditions (2′), and the boundary conditions

$$\sigma_{yy} = 0, \quad \sigma_{xy} = 0 \quad \text{when} \quad -h \le x \le h, \quad y = l;$$

$$\sigma_{xy} = 0, \quad v = 0 \quad \text{when} \quad -h \le x \le h, \quad y = 0;$$

$$\sigma_{xy} = 0 \quad \text{when} \quad x = \pm h, \quad 0 \le y \le l;$$

$$\sigma_{xx} = (1 - 2c^2)\, \rho c_1^2 \frac{w_0}{c_1} \quad \text{when} \quad x = \pm h, \quad \frac{y}{c_1} < t < \frac{2l - y}{c_1};$$

$$\sigma_{xx} = 0 \quad \text{when} \quad 0 < t < \frac{y}{c_1}, \quad \frac{2l - y}{c_1} < t < \frac{2l}{c_1}.$$

The initial system of equations and the boundary conditions involve the following dimensionless governing parameters of the problem:

$$c = \frac{c_2}{c_1}, \quad w_0' = \frac{w_0}{c_1}, \quad h' = \frac{h}{l}. \qquad (18)$$

By virtue of the linearity of the problem and the fact that the velocity of propagation of disturbances does not exceed c_1, the total force along the y axis at $y = 0$ can be calculated from formula (14) provided that: 1) $0 < t < 2l/c_1$, where $2l/c_1$ is the time of arrival of the reflected wave from the free upper end of the rod $y = l$; 2) $h < l$, i.e., the dimensions of the rod are such that the time of passage of the wave in the transverse direction exceeds the time of passage in the longitudinal direction.

As is known, the change in the momentum of a body is equal to the impulse of the external forces. Here, the only external force affecting the rod is the total force at the boundary of contact of the rod and the obstacle.

Then, the total change in the momentum of the body is

$$\Delta P = 2h\rho c_1^2 \frac{w_0}{c_1} \int_0^{\frac{2l}{c_1}} dt - k(c) \frac{4}{\pi} \rho c_1^2 w_0 (1 - 2c^2)^2 \int_0^{\frac{2l}{c_1}} t dt,$$

$$\Delta P = \left(\rho 4lh - \frac{8}{\pi} \rho l^2 k(c)(1 - 2c^2) \right) w_0 .$$

The initial momentum is

$$P_1 = -\rho 2lhw_0 , \quad |P_1| = \rho 2lhw_0 .$$

The momentum of the reflected object is

$$P_2 = \rho 2lhw_0 - \frac{8}{\pi}(1 - 2c^2)^2 \rho l^2 k(c)\, w_0 .$$

The defect (loss) of momentum is

$$DP = |P_1| - P_2 = \frac{8}{\pi}(1 - 2c^2)^2 k(c) \rho l^2 w_0 ,$$

and the relative momentum defect is

$$DP' = \frac{DP}{|P_1|} = \frac{4}{\pi} \frac{(1 - 2c^2)^2 k(c)}{h'} . \tag{19}$$

This formula is obtained under the assumption that $h' > 1$.

As is seen from relation (19), the relative momentum defect depends on just two dimensionless determining parameters of the problem from (18) – c and h' – and does not depend on the initial velocity w_0'.

We note that in the derivation of formula (19) it was implicitly assumed that the stresses σ_{yy} at the boundary $y = 0$ are compressive. However, assuming that $h = l$ and thus $c \to 0$, we obtain $DP/P_1 = 4/\pi > 1$, which leads to an obvious contradiction. This is caused by the fact that disruption of the contact between the rod and the obstacle does not always occur over the entire surface of contact upon the arrival of the wave reflected from the upper end $z = l$, but it can begin as a result of the arrival of disturbances from the boundary $x = \pm h$. Loss of contact begins at the center of the rod (at $x = 0$) upon superposition of waves that came from the ends $x = \pm h$. The time of the beginning of contact loss is determined by the equation

$$-\rho c_1^2 \frac{w_0}{c} + 2\sigma_{yy}(h; 0; t) = 0 ,$$

which in explicit form is represented as

$$-1 + 2(1 - 2c^2) \frac{4}{\pi} \left[g_1\left(\frac{h}{c_1 t}, c \right) + g_2\left(\frac{h}{c_1 t}, c \right) \right] = 0 , \tag{20}$$

where

$$g_1\left(\frac{h}{c_1 t}, c \right) = 0$$

when $h > c_1 t$;

$$g_1\left(\frac{h}{c_1 t}, c \right) = \int_{h/(c_1 t)}^{1} \frac{[u^2 - 2c^2][u^2(1 - 2c^2) + 2c^2]}{[u^2(1 - 2c^2) + 2c^2]^2 - 4c^3(1 - u^2)\sqrt{u^2(1 - c^2) + c^2}} \frac{du}{\sqrt{1 - u^2}}$$

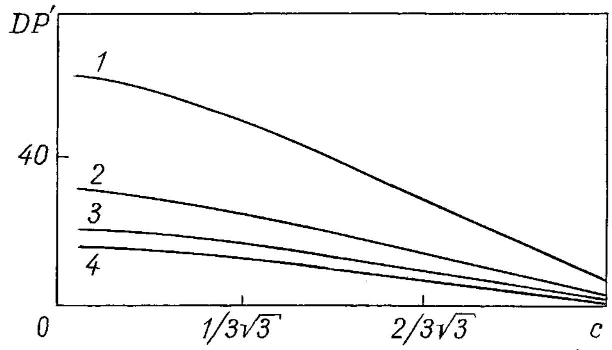

Fig. 2. Dependences of the relative momentum defect DP' on c for four variants (curves 1, 2, 3, and 4 correspond to $h' = 2$, 4, 6, and 8, respectively). DP', %.

when $h < c_1t < 2h$;

$$g_2\left(\frac{h}{c_1 t}, c\right) = 0$$

when $h > cc_1t$;

$$g_2\left(\frac{h}{c_1 t}, c\right) = \int_{h/(c_1t)}^{c} \frac{4c^4\sqrt{c^2 - u^2}}{[2c^2 - u^2]^2 - 4c^2(c^2 - u^2)\sqrt{c^2 - (1 - c^2)u^2}} \frac{du}{\sqrt{c^2 - (1 - c^2)u^2}}$$

when $h < cc_1t < 2h$.

Considering h/c_1t as the unknown quantity in Eq. (20), we denote its solution by $z^*(c)$. Then formula (19) can be used for the conditions

$$h > l, \quad \frac{2l}{c_1} < \frac{h}{c_1 z^*(c)}$$

or in dimensionless variables for

$$h' > 1, \quad 2z^* < h'.$$

We note that formula (19) is obviously applicable when $h > 2l$, i.e., $h' > 2$.

The problem of the effect of the inhomogeneity of the rod on the momentum defect in impact against a rigid obstacle due to losses to longitudinal oscillations was studied numerically in [4].

Figure 2 presents dependences of the relative defect (loss) of momentum $DP' = DP/|P_1|$ on the parameter $c = c_2/c_1$ for four strikers for variation of the parameter $h' = h/l$ from 2 to 8. Here, the relative momentum losses decrease within the half-open interval $]0, 1/\sqrt{3}]$ from 62 to 8.2% (at $h' = 2$), from 31 to 4.1% (at $h' = 4$), from 21 to 2.7% (at $h' = 6$), and from 15.8 to 2.1% (at $h' = 8$). As the transverse dimensions of the striker h increase (the longitudinal dimension l being fixed), momentum losses due to oscillations naturally decrease (see curves 1–4 in Fig. 2).

Figure 3 shows hyperbolic dependences of the relative momentum defect DP' on the geometric parameter h' for variation of it within the half-open interval $]0, 10]$ for three homogeneous strikers made of different metals. It is seen that with increase in the parameter c the momentum losses decrease.

In conclusion we note that during the theoretical analysis we revealed and analytically estimated (in the form of a finite relation) the loss of momentum by a short elastic rod in longitudinal impact against a

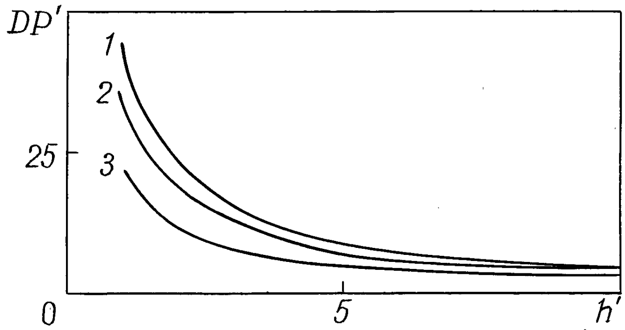

Fig. 3. Dependences of the relative momentum defect DP' on h' for different materials of the striker (curves 1, 2, and 3 correspond to aluminum (c = 0.46), copper (c = 0.49), and steel (c = 0.55), respectively).

rigid obstacle due to the presence of transverse waves. It is found that disruption of contact between the striker and the obstacle for certain relations between its transverse and longitudinal dimensions can begin even before the arrival of the wave reflected from the upper end; this loss of contact can be initiated by reflected waves that came from the side boundaries, and it begins at the center of the contact area in the interference of the waves.

This work was carried out with financial support from the Russian Fund for Fundamental Research, grant No. 98-01-00019.

NOTATION

u and v, displacements along the Cartesian coordinates x and y, respectively; c_1 and c_2, velocities of propagation of the volume and shear waves, respectively; σ_{yy}, σ_{xy}, and σ_{xx}, stresses; λ and μ, Lamé constants; t, time; ρ, material density; ε_{yy}, ε_{xy}, and ε_{xx}, deformations; $f(y, t)$, total force along the y axis at the cross section y = const; h and l, vertical and horizontal dimensions of the rod; ω, parameter of the Fourier transformation; w_0, collision velocity; P_1 and P_2, initial momentum and momentum of reflection; ΔP, change in momentum.

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